

OCR

Oxford Cambridge and RSA

Wednesday 7 June 2017 – Morning

AS GCE MATHEMATICS

4732/01 Probability & Statistics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

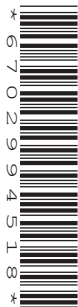
OCR supplied materials:

- Printed Answer Book 4732/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

- 1 A researcher investigates the yield, y kilograms, of a process when different amounts, x grams, of a certain additive are used. The results are shown below.

x	5	6	7	8	9	10	11
y	133.2	133.2	133.5	133.6	133.9	133.8	134.0

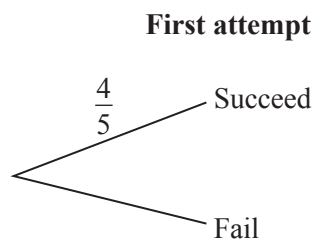
$$n = 7 \quad \Sigma x = 56 \quad \Sigma y = 935.2 \quad \Sigma x^2 = 476 \quad \Sigma y^2 = 124\,943.34 \quad \Sigma xy = 7485.6$$

- (i) Calculate the product moment correlation coefficient, r . [3]
- (ii) State the effect, if any, on the value of r if the mass of the yield were measured in tonnes instead of kilograms. [1]
- (iii) Calculate the equation of the regression line of y on x . [3]
- (iv) It is required to estimate the amount of additive needed to give a certain yield. Give a reason why the regression line of y on x should be used, rather than the regression line of x on y . [1]
- 2 In a high jump competition, jumpers are allowed three attempts to succeed at each height. For one particular height Imran estimates his chances of succeeding as follows.

- The probability that he will succeed on his first attempt is $\frac{4}{5}$.
- If he fails on his first attempt, the probability that he will succeed on his second attempt is $\frac{3}{4}$.
- If he fails on his first two attempts, the probability that he will succeed on his third attempt is p .

Use Imran's estimates to answer the following.

- (i) In your Printed Answer Book complete the probability tree diagram for this situation. [2]



- (ii) Find the probability that Imran succeeds on either his first or his second attempt. [3]
- (iii) Given that the probability that Imran succeeds at this particular height is $\frac{197}{200}$, find p . [3]

3 A bag contains 6 red discs and 4 green discs.

(i) Rohan takes 3 discs at random from the bag, without replacement, and places them on the table. Let X be the number of red discs he places on the table.

(a) Show that $P(X = 1) = \frac{3}{10}$. [3]

(b) Part of the probability distribution of X is given in the table below.

x	0	1	2	3
$P(X = x)$	$\frac{1}{30}$	$\frac{3}{10}$	$\frac{1}{2}$	

Find $E(X)$ and $\text{Var}(X)$. [6]

(ii) Rohan arranges the 6 red discs and 4 green discs in a straight line. How many different arrangements of the colours are possible? [2]

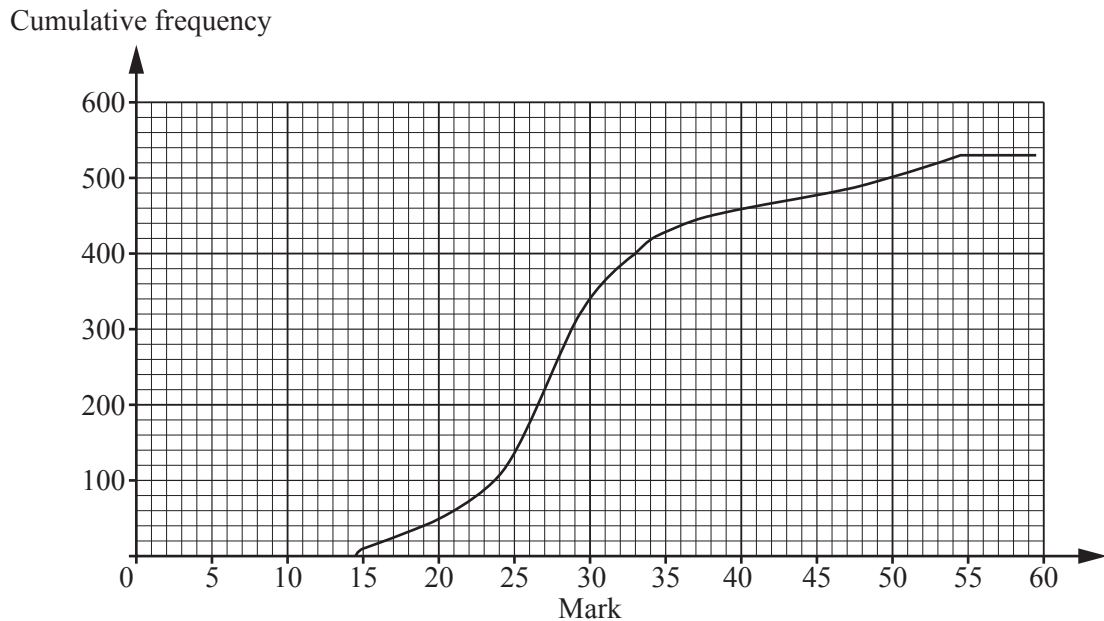
4 In a class of 30 students, each student studies exactly one modern language. 14 students study French, 9 students study Spanish and 7 students study German. A committee of 6 students is to be chosen from these 30 students. Find the number of ways of choosing the committee if it contains

(i) any 6 students from the class, [1]

(ii) 2 students studying each language, [2]

(iii) exactly 1 student studying French. [3]

- 5 The marks of some students in an examination were summarised in a grouped frequency distribution, using the following classes: 10–14, 15–19, 20–24, 25–29, 30–34, 35–39, 40–44, 45–49, 50–54, 55–59, all inclusive. A cumulative frequency diagram was drawn, as shown below.



- (i) How many students took the examination? [1]
- (ii) 20% of students gained the top grade. Find the minimum mark for the top grade. [3]
- (iii) A teacher said
- “The cumulative frequency graph shows that the highest mark scored by any student was 54 or 55.”
- Explain why this statement is incorrect, and give an improved statement about the highest mark. [2]
- (iv) State which class is the modal class, explaining how you know. [2]

- 6 Two judges gave the following scores to five contestants.

Contestant	Alan	Jo	Mike	Steve	Mary
Judge 1	5.8	5.5	5.1	4.7	3.9
Judge 2	5.4	5.9	5.1	4.9	4.5

- (i) In your Printed Answer Book, complete the table of ranks. [2]
- (ii) Calculate Spearman's rank correlation coefficient, r_s . [3]
- (iii) Another two judges also ranked the same five contestants. These judges were inexperienced, so that their ranks can be regarded as random. Find the probability that the value of r_s for these two judges' ranks will be the same as that for judges 1 and 2. [4]
- 7 The mean and standard deviation of the weights, w grams, of a sample of 75 stones were found to be 52.3 and 5.8 respectively.

- (i) Find the value of Σw^2 . [2]

The weights, x grams, of another sample of 100 stones were found and were summarised as follows.

$$n = 100 \qquad \Sigma x = 5760 \qquad \Sigma x^2 = 335497$$

- (ii) Calculate the mean and standard deviation of the weights of all 175 stones. [4]
- 8 Every month Frankie posts 10 parcels, one to each of 10 friends. The friends live in different towns in a country where the postal service is unreliable. She has found that, for each parcel, the probability that it arrives is $\frac{7}{8}$.
- (i) Name an appropriate distribution with which to model the number of parcels that arrive in a particular month, giving the value(s) of any parameters. State a necessary assumption for the model to be valid. [2]
- (ii) Use this model to find the probability that in a particular month
- (a) all 10 parcels arrive, [1]
- (b) at least 9 parcels arrive. [2]
- (iii) Frankie chooses 5 months at random. Find the probability that all 10 parcels arrive in at least 4 of these 5 months. [3]

- 9 A firm that manufactures chocolate bars gives away vouchers for a free tour of their factory. Vouchers are placed at random in the wrappers of 20% of the chocolate bars made by the firm. No wrapper contains more than one voucher. Charlie buys exactly one chocolate bar each day.

(i) Find the probability that he finds his first voucher

(a) on the 4th day, [2]

(b) after the 4th day. [2]

Each voucher allows one person to take the tour. There are 5 people in Charlie's family, including Charlie.

(ii) Find the probability that he has to buy exactly 10 chocolate bars in order to have enough vouchers for his whole family. [4]

S1 June 2017 Mark Scheme SSU v4

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to > 3sfs, ISW for later rounding. Penalise over-rounding only once in paper.

Question		Answer	Mk	Guidance	
1	i	$S_{xx} = 476 - \frac{56^2}{7} \quad (= 28)$	M1	Correct method for one S	
		$S_{yy} = 124943.34 - \frac{935.2^2}{7} \quad (= 0.62)$			
		$S_{xy} = 7485.6 - \frac{56 \times 935.2}{7} \quad (= 4)$			
		$r = \frac{"4"}{\sqrt{"28" \times "0.62"}}$			
		= 0.960 (3 sf)	A1	allow 0.96	Correct ans, no wking, M1M1A1
	ii	None oe	B1 [1]		Ignore all else
	iii	$b = \frac{"4"}{"28"} \quad (= \frac{1}{7} \text{ or } 0.14 \text{ or better})$	M1	ft their Ss from (i) for M1M1 not A1	or $b = \frac{7485.6 - \frac{56 \times 935.2}{7}}{476 - \frac{56^2}{7}}$
		$y - \frac{935.2}{7} = \frac{1}{7} (x - \frac{56}{7})$ oe	M1	or $a = \frac{935.2}{7} - \frac{1}{7} \times \frac{56}{7}$ oe	or $a = 133.6 - \frac{1}{7} \times 8$
		$y = 0.143x + 132$ or $y = \frac{1}{7}x + \frac{4636}{35}$	A1	oe Correct to 3 sfs except allow 132.5 Must include "y =" for A1	but allow $y = 0.14x + 130$ with no error seen Correct ans, no wking, M1M1A1
	iv	x is controlled Allow x is independent or Amount of additive is controlled	B1 [1]	or values of x are fixed, given, exact, or x is changed NOT "x changes" or "x is constant" NOT "x is known"	Ignore all else NOT x doesn't depend on y NOT y depends on x or y is depend't NOT "x increases by same amount each time"
2	i	All correct lines & probs OR labels	B1	Allow extra lines with no probs given, or prob = 0 given, for B1B1	"probs" includes $1 - p$ Ignore products at end, if shown
		All correct lines & probs & labels	B1 [2]		

				SC: One line omitted, all probs and labels given on other lines B1B0	NOT q instead of $1 - p$
	ii		$\frac{4}{5} + \frac{1}{5} \times \frac{3}{4} \quad \text{or } 1 - \frac{1}{5} \times \frac{1}{4}$ $= \frac{19}{20} \text{ or } 0.95$	M2 $\frac{4}{5}$ +prod of 2 P's or 1- prod of 2 P's M1 A1 No ft from tree diag. [3]	eg $\frac{4}{5} + \frac{1}{5} \times \frac{4}{5}$ or $1 - \frac{1}{5} \times \frac{1}{5}$ or $\frac{4}{5} + \frac{1}{5} \times \frac{3}{5}$ or $1 - \frac{1}{5} \times \frac{2}{5}$ M1M0A0
	iii		$1 - \frac{1}{5} \times \frac{1}{4} \times (1-p) = \frac{197}{200} \quad \text{or } \frac{3}{200} \text{ seen}$ $\frac{1-p}{20} = \frac{3}{200} \text{ any correct step, one fract each side}$ $p = \frac{7}{10}$	M1 or '0.95'+ $\frac{1}{5} \times \frac{1}{4} \times p = \frac{197}{200}$ or $\frac{7}{200}$ seen M1d eg $\frac{19+p}{20} = \frac{197}{200}$ or $\frac{1}{20} p = \frac{7}{200}$ Dep 1st M1 A1 $\frac{197}{200} - \left(\frac{4}{5} + \frac{1}{5} \times \frac{3}{4}\right) \quad (= \frac{7}{200}) \quad \text{M1}$ $\frac{7}{200} \div \left(\frac{1}{4} \times \frac{1}{5}\right) \text{ or } \frac{7}{200} \times 20 \text{ oe M1}$ [3] $= \frac{7}{10}$ A1	or $\frac{4}{5} + \frac{1}{5} \times \frac{3}{4} + \frac{1}{5} \times \frac{1}{4} \times p = \frac{197}{200}$ eg $\frac{1}{20} p = \frac{7}{200}$ oe in decimals ft from tree diag for M1M1, not A1 or similar arithmetic methods
3	i	a	$\frac{6}{10} \times \frac{4}{9} \times \frac{3}{8} \text{ oe}$ $\times 3$ $= \frac{3}{10} \text{ oe AG}$	M1 Must see this, oe M1 prod of any 3 probs $\times 3$ or add 3 prods of 3 probs A1 NB Incorrect methods = $\frac{3}{10}$ M0M0A0: eg $\frac{\text{No. of discs taken}}{\text{Total no. of discs}} = \frac{3}{10}$ eg $1 - \left(\frac{1}{30} + \frac{3}{30} + \frac{1}{2} + \frac{1}{6}\right) = \frac{3}{10}$ eg $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$ with no other wking [3]	${}^6C_1 \times {}^4C_2$ (must see 4C_2) M1 $\div {}^{10}C_3$ any no. $\div {}^{10}C_3$ or 120 M1 NB ${}^3C_2 \times 0.6 \times 0.4^2$ scores M0M1A0

	i	b	$P(X = 3) = \frac{1}{6}$ or $\frac{5}{30}$ oe or 0.167 (3 sf) Σxp $= \frac{9}{5}$ or $1\frac{4}{5}$ or 1.8 oe $\Sigma x^2 p$ (= 3.8) $- "1.8" ^2$ $= \frac{14}{25}$ or 0.56 oe	B1 M1 A1ft M1 M1 A1 [6]	May be seen in table or workng ≥ 2 non-zero terms correct, ft their $\frac{1}{6}$ If $\div 4$: M0 ft their $\frac{1}{6}$ ≥ 2 non-zero terms correct, ft their $\frac{1}{6}$. If $\div 4$: M0 any no – their μ^2 , dep +ve result cao	May be implied by ans to mean $(x - "1.8")$ attempted all 4 values M1 $\Sigma(x - "1.8")^2 p \geq 3$ terms correct M1
	ii		$\frac{10!}{4! \times 6!}$ or ${}^{10}C_4$ or ${}^{10}C_6$ alone $= 210$	M1 A1 [2]	$210 \times \dots$ or $\div \dots$ M0A0	
4	If P used instead of C <u>consistently in all parts attempted</u> (at least two parts attempted), max marks: (i) B0 (ii) M1A0 (iii) M1M1A0 Answers: (i) 427518000 (ii) 550368 (iii) 7338240					
4	i		593775	B1 [1]	or 594000 (3 sf)	
	ii		${}^{14}C_2 \times {}^9C_2 \times {}^7C_2$ alone $= 68796$	M1 A1 [2]	or 68800 (3 sf)	MR: $\div {}^{30}C_6$ ($= \frac{84}{725}$ or 0.116) M1A0
	iii		14 (or ${}^{14}C_1$) $\times {}^{16}C_5$ or 14×4368 alone $= 61152$	M2 A1 [3]	or M1 for either ${}^{16}C_5$ or 4368 seen or 14 (or ${}^{14}C_1$) \times any no. seen or 61200 (3 sf)	$14 \times ({}^9C_5 + {}^9C_4 \times 7 + {}^9C_3 \times {}^7C_2 + {}^9C_2 \times {}^7C_3 + 9 \times {}^7C_4 + {}^7C_5)$ M2 NOT $14 + \dots$: M0M0 MR: $\div {}^{30}C_6$ ($= \frac{224}{2175}$ or 0.103) M2A0
5	i		530 (± 5)	B1 [1]		

		$= \frac{9}{10}$ oe	A1		
	iii	$\Sigma d^2 =$ their '2' stated or implied	[3]		
		4 possible sets of ranks (Not "4" seen)	B1	eg by a set of ranks for which $\Sigma d^2 = '2'$ (could be the original set) or by two 1's and three 0's seen	or swap 2 <u>adjacent</u> ranks, stated or shown B1
		"4" \div 5!	B1		$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2}$ (\times ..but not squared) M1
		$= \frac{1}{30}$ oe or 0.0333 (3 sf)	M1	Divide any no. by 5! or 120 or 5P_3 or div by 5! \times ...	$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 4$ correct B1
			A1	but not div by $(5!)^2$ except 3rd SC below	$= \frac{1}{30}$ oe or 0.0333 (3 sf) A1
				eg $\frac{4}{5!} \times 2 = \frac{1}{15}$ B1B1M1A0	
			[4]	SC: $\frac{8}{2 \times 5!}$ or $\frac{8}{240} = \frac{1}{30}$ B1B1M1A1	
				SC: $\frac{4 \times 5!}{5!^2} = \frac{1}{30}$ B1B1M1A1	
7	i	$5.8^2 = \frac{\Sigma W^2}{75} - 52.3^2$	M1	or $5.8 = \sqrt{\left(\frac{\Sigma W^2}{75} - 52.3^2\right)}$	
		$\Sigma W^2 = 207669.75$ or $\frac{830679}{4}$ oe	A1	Allow 208000 with correct working, no errors seen	NOT other ans that rounds to 208000
			[2]		
	ii	mean = $\frac{75 \times 52.3 + 5760}{75 + 100}$	M1	or $\frac{3922.5 + 5760}{175}$ or $\frac{9682.5}{175}$	
		= 55.3 (3 sf)	A1		
		var = $\frac{207\ 669.75 + 335\ 497}{75 + 100} - 55.329^2$	M1	or $\frac{543166.75}{175} - 55.329^2$	$\frac{\text{Their(i)} + 335\ 497}{75 + 100} - (\text{their mean of } 175)^2$
		(= 42.5.....)			
		sd = 6.52 (3 sf)	A1	Allow 6.51 art 6.52 or 6.51	NB ans 6.76 prob'y from mean = 55.3 M1A1M1A0 but check wking

				[4]		NB May see 55.3 used in sd calc'n, but correct sd given (6.52). This gets full marks on the assumption that although candidate wrote "55.3" she used more sig figs in the calc'n
8	i		$B(10, \frac{7}{8})$ or Binomial & $n = 10, p = \frac{7}{8}$ Arrival of each parcel is independent or Prob parcel arrives not affected by others or Prob parcel arrives is constant oe	B1 B1 [2]	or Binomial and $(10, \frac{7}{8})$ Allow: Parcels are independent Deliveries are independent Arrivals are independent P(parcel arrives) is independent Friends are indep	NB just 10 & $\frac{7}{8}$ seen: not enough In context Ignore all else The two B-marks are independent NOT No other factors involved
	ii	a	0.263 (3 sf)	B1 [1]		
	ii	b	$P(X = 9, 10)$ $= 10(\frac{1}{8})(\frac{7}{8})^9 + (\frac{7}{8})^{10}$ alone $= 0.639$ (3 sf)	M1 A1 [2]	all correct or (ii)(a) + $10(\frac{1}{8})(\frac{7}{8})^9$ cao	or $1 - P(X \leq 8)$ all terms correct or $1 - 0.361$ 0.639, no wking, M1A1 Use of tables: M0A0 0.64, no wking: M0A0
	iii		Their "0.263" or $(\frac{7}{8})^{10}$ used $5 \times "0.263"^4 \times (1 - "0.263") + "0.263"^{10}$ $= 0.0189$ (3 sf)	M1 M1 A1 [3]	or better cao	or $1 - (0.737^5 + \dots + {}^5C_3 \times 0.737^2 \times 0.263^3)$ all 4 terms correct ft their 0.263 If (ii)(b) used instead of (ii)(a), (must see working) allow M0M1A0
9	i	a	$(1 - 0.2)^3 \times 0.2$ $= \frac{64}{625}$ or 0.102 (3 sf)	M1 A1 [2]		
	i	b	$(1 - 0.2)^4$ or $(\frac{4}{5})^4$ alone $= \frac{256}{625}$ or 0.410 (3 sf)	M1 A1 [2]	$1 - (0.2 + 0.8 \times 0.2 + 0.8^2 \times 0.2 + 0.8^3 \times 0.2)$ or $1 - (0.2 + 0.8 \times 0.2 + 0.8^2 \times 0.2 + (i)(a))$ oe allow 0.41	eg $1 - (\frac{4}{5})^4 = 0.590$ M0A0

	ii	<p>Binomial with $n = 9$ or 10 and $r > 1$</p> <p>${}^9C_4 \times (1 - 0.2)^5 \times 0.2^4$ or 0.06606 or $0.9804 - 0.9144$ or 0.066</p> <p>${}^9C_4 \times (1 - 0.2)^5 \times 0.2^4 \times 0.2$ or ${}^9C_4 \times (1 - 0.2)^5 \times 0.2^5$</p> <p>or $(0.9804 - 0.9144) \times 0.2$</p> <p>$= 0.0132$ (3 sf) or $\frac{129024}{9765625}$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>eg by ${}^9 \text{ or } {}^{10}C_r$ ($r > 1$) or $p^a \times (1 - p)^b$ ($a + b = 9$ or 10 and $a, b > 1$)</p> <p>or attempt $P(4 \text{ vouchers in } 9) \times 0.2$ eg $0.8^5 \times 0.2^4 \times 0.2$ or $0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2$</p> <p>Fully correct method</p>	<p>or use of bin table for $n = 9$ or 10 eg 0.9936 or 0.9672</p> <p>but NOT just $0.8^5 \times 0.2^5$</p> <p>Examples:</p> <p>$0.8^5 \times 0.2^4 \times 0.2$ M1M1A0A0 $0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2$ M1M1A0A0 0.066 or better M1M1A0A0</p> <p>${}^{10}C_5 \times 0.8^5 \times 0.2^5$ M1M0M0A0 $0.9936 - 0.9672$ M1M0M0A0 $0.8^5 \times 0.2^5$ M1M0M0A0</p>
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Total 72 marks